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Ghost sector of vacuum string field theory and the projection equation

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Abstract: We study the ghost sector of vacuum string field theory where the BRST operator Q is given by the midpoint insertion proposed by Gaiotto, Rastelli, Sen and Zwiebach. We introduce a convenient basis of half-string modes in terms of which Q takes a particularly simple form. We show that there exists a field redefinition which reduces the ghost sector field equation to a pure projection equation for string fields satisfying the constraint that the ghost number is equally divided over the left- and right halves of the string. When this constraint is imposed, vacuum string field theory can be reformulated as a U(∞) cubic matrix model. Ghost sector solutions can be constructed from projection operators on half-string Hilbert space just as in the matter sector. We construct the ghost sector equivalent of various well-known matter sector projectors such as the sliver, butterfly and nothing states.

Keywords: Bosonic Strings, Tachyon Condensation, String Field Theory.

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1. Introduction

Witten's open string field theory (OSFT) [1] has proven succesful in desribing the process of tachyon condensation on unstable brane configurations. The solution corresponding to the tachyon vacuum has been approximated to a high degree of accuracy using level truncation methods [2]. However, an explicit analytic solution is still lacking. Such a solution would be desirable in order to describe the fluctuations around the tachyon vacuum, where interesting

new physics is expected to emerge, possibly including closed strings [3]. Vacuum string field theory (VSFT) was proposed as an 'educated guess' for the string field theory around the tachyonic vacuum. Various requirements for such a theory led the authors of [4] to propose an action of the same cubic form as in OSFT but with a different BRST-operator Q depending only on the ghost modes. This makes it natural to look for solutions Ψ in which the matter and ghost parts decouple:

$$\Psi = \Psi^m \otimes \Psi^{gh} \,. \tag{1.1}$$

The equation of motion for the matter part then becomes the projection equation

$$\Psi^m = \Psi^m \star^m \Psi^m \tag{1.2}$$

while the ghost part has to satisfy

$$Q\Psi^{gh} = \Psi^{gh} \star^{gh} \Psi^{gh} \,. \tag{1.3}$$

Various checks of VSFT can be made without precise knowledge of Q or the ghost part Ψ^{gh}. The matter part of the D25-brane solution is given by the 'sliver state' [5] and lower dimensional D-branes correspond to generalizations thereof. These solutions reproduce the correct ratios of D-brane tensions [6, 7]. Multi-D-brane solutions have also been constructed [8].

A convenient formulation for finding solutions to the projection equation (1.2) is given in the split-string formalism [11, 12, 13]. Here, it is possible to represent string fields as operators on an auxiliary 'half-string Hilbert space' $\mathcal{H}^{\frac{1}{2}}$ in such a way that the star product becomes the multiplication of operators. Numerous solutions to (1.2) can then be constructed in the form of projection operators on $\mathcal{H}^{\frac{1}{2}}$. This construction generalizes a familiar technique from noncommutative field theories [14]. Another approach to constructing solutions to (1.2) uses surface states in conformal field theory [9, 10, 18].

In [16], Gaiotto, Rastelli, Sen and Zwiebach (GRSZ) presented convincing arguments that point to a specific choice for the BRST operator Q in VSFT. It is given by a pure midpoint insertion

$$\begin{aligned} Q &= (c_0 - (c_2 + c_{-2}) + (c_4 + c_{-4}) + \cdots) \\ &= \frac{1}{2}\left(c^+\left(\frac{\pi}{2}\right) + c^-\left(\frac{\pi}{2}\right)\right). \end{aligned} \tag{1.4}$$

It was also argued that the overall multiplication constant in front of the action should diverge in order to have finite energy solutions.

Finding solutions to the ghost sector equation (1.3) of VSFT seems at first sight more difficult than in the matter sector because of the presence of Q and the fact that $\star^{gh}$ contains a midpoint insertion. However, solutions were found in [16] which correspond to projectors in an auxiliary twisted ghost system. In this paper we further explore the relation between solutions to the ghost sector equation (1.3) with Q given by the midpoint insertion (1.4) and projection operators. Our approach uses oscillator methods [19, 20] and the bosonized form of the ghosts. After transforming to suitably chosen half-string

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variables, we show that there exists a field redefinition which takes the ghost equation of motion into a pure projection equation of the form

$$\Psi' = \Psi' \star' \Psi' \,. \tag{1.5}$$

provided that Ψ' satisfies the constraint that the ghost number should be equally divided over the left- and right halves of the string. The star product $\star'$ in (1.5) is different from the original ghost sector product $\star^{gh}$ in that it no longer includes midpoint insertions: it is given by a pure delta-function overlap just as in the matter sector. This allows us to construct solutions from projection operators on half string Hilbert space as in the matter sector. In this way several well-known matter sector solutions have a direct counterpart in the ghost sector. As an illustration, we construct the ghost sector equivalent of various well-known matter sector projectors such as the sliver, butterfly and nothing states.

This paper is organized as follows. Section 2 deals with the transformation to half-string modes which we illustrate in the matter sector. We are careful to choose a half-string basis which gives well defined expressions for midpoint insertions. We discuss the Bogoliubov transformation relating full- and half-string oscillator modes and derive the relation between full-string and half-string vacua. We also find expressions for the Neumann matrices in the half-string basis. In section 3, we discuss several solutions to the matter sector projection equation which will turn out to have a direct counterpart in the ghost sector. In section 4, we turn to the transformation to half-string variables in the ghost sector. We derive the bosonized version of the VSFT BRST-operator which takes a simple form in terms of our half-string variables. In section 5, we discuss the field redefinition which turns the ghost sector equation of motion into a pure projection equation for string fields satisfying the constraint that the ghost number is equally divided into the left and right-parts. When this constraint is imposed, vacuum string field theory can be reformulated as a U(∞) cubic matrix model. We comment on the construction of solutions and give several examples. We end with a discussion of some open problems.

2. Vacuum string field theory: matter sector

2.1 Mode expansions

We start this section by giving some conventions regarding mode expansions. The string is parametrized by $\sigma \in [0, \pi]$ and mode expansions are obtained by expanding in orthonormal basis functions $\psi_n(\sigma)$ of $L_2[0, \pi]$ obeying Neumann boundary conditions at the endpoints:

$$\psi_0 = 1\,; \qquad \psi_n = \sqrt{2}\,\cos n\sigma\,. \tag{2.1}$$

One obtains position and momentum modes by expanding (we will suppress Lorentz indices in the rest of the paper):

$$\begin{aligned} X(\sigma) &= \sum_{n=0}^{\infty} x_n \psi_n(\sigma) = x_0 + \sqrt{2}\sum_{n=1}^{\infty} x_n \cos n\sigma \\ \pi P(\sigma) &= \sum_{n=0}^{\infty} p_n \psi_n(\sigma) = p_0 + \sqrt{2}\sum_{n=1}^{\infty} p_n \cos n\sigma\,. \end{aligned}$$

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We represent the modes $\{x_n\}_n$ as a column vector $|x)$ and use the notation $(x|y) \equiv \sum_{n=0}^{\infty} x_n y_n$. Creation-annihilation operators are defined by:

$$\begin{aligned} |x) &= \frac{i}{\sqrt{2}} E[|a) - |a^\dagger)] & |p) &= \frac{1}{\sqrt{2}} E^{-1}[|a) + |a^\dagger)] \\ |a) &= \frac{1}{\sqrt{2}}[E|p) - iE^{-1}|x)] & |a^\dagger) &= \frac{1}{\sqrt{2}}[E|p) + iE^{-1}|x)] \end{aligned} \tag{2.2}$$

with E the matrix with coefficients

$$E^{-1}_{mn} = \sqrt{n}\,\delta_{mn} + \sqrt{2}\,\delta_{m0}\delta_{n0}\,.$$

The canonical commutation relations are:

$$[a_m, a_n^\dagger] = \delta_{mn}\,.$$

As usual, one builds up a Fock space by acting with the creation operators on a vacuum state $|\Omega\rangle$ annihilated by all a_n (including a_0) and normalized to $\langle\Omega|\Omega\rangle = 1$. Sometimes it is useful to work with the translationally invariant vacuum $|0\rangle$ satisfying $p_0|0\rangle = a_n|0\rangle = 0,\ n > 0$. Position basis states can be expressed in terms of oscillators as

$$|x\rangle \equiv |\{x_n\}_n\rangle = N^X \exp\left[-\frac{1}{2}(x|E^{-2}|x) - \sqrt{2}\,i(a^\dagger|E^{-1}|x) + \frac{1}{2}(a^\dagger|a^\dagger)\right]|\Omega\rangle \tag{2.3}$$

with N^X an (infinite) normalization constant:

$$N^X = \left(\frac{2}{\pi}\right)^{26/4} \prod_{n=1}^{\infty} \left(\frac{n}{\pi}\right)^{26/4}.$$

2.2 Split-string formalism

In this section we set up the transition to the split-string formalism. This transition is usually [11, 12] obtained by splitting $X(\sigma)$ into left and right parts

$$\begin{aligned} X^L(\sigma) &= X(\sigma) \qquad 0 \le \sigma \le \frac{\pi}{2} \\ X^R(\sigma) &= X(\pi - \sigma) \qquad 0 \le \sigma \le \frac{\pi}{2} \end{aligned} \tag{2.4}$$

and performing mode expansions of these functions on the interval $[0, \pi/2]$, where one chooses some boundary condition in the midpoint $\sigma = \pi/2$. However, this leaves some ambiguities in the treatment of the midpoint: for instance, it is not clear how to express $X(\pi/2)$ in terms of half-string modes. Our setup is slightly different in the sense that we will treat the transition to half-string modes as a change of basis *in the full interval* $[0, \pi]$. The change of basis is chosen so as to diagonalize the projection operators on the left and right halves of the string. Various choices of orthonormal half-string bases are possible and the expansion of a string configuration $X(\sigma)$ in any such basis will converge with respect to the L_2 norm. However, even if $X(\sigma)$ is continuous in $\sigma = \pi/2$, pointwise convergence of the expansion in $\sigma = \pi/2$ is not guaranteed. Due to the importance of the midpoint in string field theory (especially in the ghost sector where midpoint insertions enter explicitly), we will impose pointwise convergence at the midpoint as an additional criterion for the half-string basis we will use. In a basis satisfying this criterion, $X(\pi/2)$ has a well-defined expansion in terms of half-string modes.

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2.2.1 Half-string projectors

Instead of working with the basis (2.1), one can also set up an expansion in a different orthonormal basis in which the projection operators P^L, P^R on the left- and right parts of the string become diagonal. The coefficients in such an expansion are referred to as half-string modes. The matrix elements of $P^L(\sigma,\sigma') = \theta(\pi/2-\sigma)\delta(\sigma-\sigma')$ with respect to the basis (2.1) are easily obtained. Splitting the component indices in even and odd ones (denoted by superscripts ^{e} and ^{o} respectively) and writing any matrix M as $M = \begin{pmatrix} M^{ee} & M^{eo} \\ M^{oe} & M^{oo} \end{pmatrix}$, one finds

$$P^L = \frac{1}{2}\begin{pmatrix} 1^{ee} & A \\ A^T & 1^{oo} \end{pmatrix}.$$

Here we have defined a matrix A whose rows are labeled by even indices and whose columns are labeled by odd ones[1]

$$\begin{aligned} A_{2m,2n+1} &= \frac{4(2n+1)(-1)^{m+n}}{((2n+1)^2-(2m)^2)\pi} \\ A_{0,2n+1} &= \frac{2\sqrt{2}\,(-1)^n}{(2n+1)\pi} \end{aligned} \tag{2.5}$$

The matrix A satisfies $AA^T = 1^{ee}$, $A^TA = 1^{oo}$, hence $(P^L)^2 = P^L$.

Similarly, for the projection operator on the right half of the string $P^R(\sigma,\sigma') = \theta(\sigma-\pi/2)\delta(\sigma-\sigma')$, one finds the component form

$$P^R = 1 - P^L = \frac{1}{2}\begin{pmatrix} 1^{ee} & -A \\ -A^T & 1^{oo} \end{pmatrix}.$$

2.2.2 Half-string basis

The Hilbert space $L_2[0,\pi]$ decomposes into the direct sum of P^L and P^R-invariant subspaces. Our goal is to introduce a new orthonormal basis adapted to this decomposition. Consider the matrix

$$\mathcal{O} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1^{ee} & A \\ 1^{ee} & -A \end{pmatrix}.$$

It is orthogonal, $\mathcal{O}\mathcal{O}^T = \mathcal{O}^T\mathcal{O} = 1$ and diagonalizes P^L, P^R:

$$\mathcal{O}P^L\mathcal{O}^T = \begin{pmatrix} 1^{ee} & 0^{ee} \\ 0^{ee} & 0^{ee} \end{pmatrix}, \qquad \mathcal{O}P^R\mathcal{O}^T = \begin{pmatrix} 0^{ee} & 0^{ee} \\ 0^{ee} & 1^{ee} \end{pmatrix}.$$

The matrix $\mathcal{O}$ defines a transformation to a new orthonormal basis $\psi^L(\sigma)$, $\psi^R(\sigma)$ satisfying $P^L\psi^L_{2n} = \psi^L_{2n}$, $P^R\psi^R_{2n} = \psi^R_{2n}$ and $P^L\psi^R_{2n} = P^R\psi^L_{2n} = 0$:

$$\psi^L_{2n}(\sigma) = \frac{1}{\sqrt{2}}[\psi_{2n}(\sigma) + \sum_m A_{2n,2m+1}\psi_{2m+1}(\sigma)] \tag{2.6}$$

$$\psi^R_{2n}(\sigma) = \frac{1}{\sqrt{2}}[\psi_{2n}(\sigma) - \sum_m A_{2n,2m+1}\psi_{2m+1}(\sigma)] \tag{2.7}$$

[1] The matrix X defined in [12] is related to our A by $X = \begin{pmatrix} 0^{ee} & A \\ A^T & 0^{oo} \end{pmatrix}$.

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Explicitly, one has

$$\begin{aligned}
\psi_0^L(\sigma) &= \sqrt{2}\,\theta\left(\frac{\pi}{2}-\sigma\right); & \psi_0^R(\sigma) &= \sqrt{2}\,\theta\left(\sigma-\frac{\pi}{2}\right)\\
\psi_{2n}^L(\sigma) &= 2\cos 2n\sigma\theta\left(\frac{\pi}{2}-\sigma\right); & \psi_{2n}^R(\sigma) &= 2\cos 2n\sigma\theta\left(\sigma-\frac{\pi}{2}\right) \qquad n\neq 0
\end{aligned} \tag{2.8}$$

where the step function in 0 is to be interpreted as $\theta(0) = 1/2$. Hence this choice of half-string basis corresponds to expanding $X^L(\sigma)$, $X^R(\sigma)$ in basis functions on $[0, \pi/2]$ obeying Neumann boundary conditions at the midpoint. The mode expansion in the new basis reads

$$X(\sigma) = \sum_{n=0}^{\infty} x_{2n}^L \psi_{2n}^L(\sigma) + \sum_{n=0}^{\infty} x_{2n}^R \psi_{2n}^R(\sigma)\,. \tag{2.9}$$

The transformation formulae for the position modes take the form:

$$|x^L) = \frac{1}{\sqrt{2}}[|x^e) + A|x^o)] \qquad |x^R) = \frac{1}{\sqrt{2}}[|x^e) - A|x^o)]\,. \tag{2.10}$$

The inverse transformations are:

$$|x^e) = \frac{1}{\sqrt{2}}[|x^L) + |x^R)] \qquad |x^o) = \frac{1}{\sqrt{2}}A^T[|x^L) - |x^R)]\,. \tag{2.11}$$

This half-string basis satisfies our criterion for pointwise convergence in the midpoint, indeed one has:

$$X(\pi/2) = \frac{1}{\sqrt{2}}(x_0^L + x_0^R) + \sum_n (-1)^n (x_{2n}^L + x_{2n}^R) = x_0 + \sqrt{2}\sum_n (-1)^n x_{2n}\,.$$

The same procedure can be followed for the decomposition of the conjugate momentum $P(\sigma)$ into half-string modes. One gets

$$\begin{aligned}
|p^L) &= \frac{1}{\sqrt{2}}[|p^e) + A|p^o)] & |p^R) &= \frac{1}{\sqrt{2}}[|p^e) - A|p^o)]\\
|p^e) &= \frac{1}{\sqrt{2}}[|p^L) + |p^R)] & |p^o) &= \frac{1}{\sqrt{2}}A^T[|p^L) - |p^R)]\,.
\end{aligned} \tag{2.12}$$

Due to the orthogonality of the transformation, the canonical commutation relations are unmodified:

$$[x_{2m}^L, p_{2n}^L] = i\delta_{mn}\,, \qquad [x_{2m}^R, p_{2n}^R] = i\delta_{mn}\,, \qquad [x_{2m}^L, p_{2n}^R] = [x_{2m}^R, p_{2n}^L] = 0\,.$$

Other half-string bases can be obtained by making further orthogonal transformations that do not mix the left- and right basis vectors. For instance, one could diagonalize the projection operators P^L, P^R with an orthogonal matrix $\tilde{\mathcal{O}}$ defined by

$$\tilde{\mathcal{O}} = \begin{pmatrix} A^T & 0^{oe} \\ 0^{eo} & A^T \end{pmatrix} \mathcal{O} = \frac{1}{\sqrt{2}} \begin{pmatrix} A^T & 1^{oo} \\ A^T & -1^{oo} \end{pmatrix}$$

The corresponding basis functions are the odd cosines

$$\tilde{\psi}_{2n+1}^L(\sigma) = 2\cos(2n+1)\sigma\,\theta\left(\frac{\pi}{2}-\sigma\right); \qquad \tilde{\psi}_{2n+1}^R(\sigma) = -2\cos(2n+1)\sigma\,\theta\left(\sigma-\frac{\pi}{2}\right).$$

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This corresponds to expanding $X^L(\sigma)$, $X^R(\sigma)$ in basis functions on $[0, \pi/2]$ obeying Dirichlet boundary conditions at the midpoint [11, 12, 13]. An expansion in this basis will give zero at $\sigma = \pi/2$, hence this half-string basis does not satisfy the criterion of pointwise convergence at the midpoint. This is often remedied by introducing an extra 'midpoint degree of freedom'. The role of such a degree of freedom is not completely clear since it doesn't correspond to a basis vector of $L_2[0, \pi]$. We will not follow this procedure here but instead use the previous half-string expansion (2.9) in the rest of the paper.

As has been noted in the literature [22, 21, 23], the matrix A is not strictly invertible, but has a (nonnormalizable) zero mode:

$$A^T_{2m+1,0}\frac{1}{\sqrt{2}} + \sum_{n=1}^{\infty} A^T_{2m+1,2n}(-1)^n = 0\,. \tag{2.13}$$

In the present context, we see from eq. (2.11) that this means that the half-string mode defined by

$$x^L_0 = -x^R_0 = \frac{1}{\sqrt{2}} \qquad x^L_{2n} = -x^R_{2n} = (-1)^n \quad (n > 0) \tag{2.14}$$

corresponds to full-string coefficients being equal to zero. It is not hard to see how this comes about. From the identity

$$1 + 2\sum_{n=1}^{\infty}(-1)^n \cos(2n\sigma) = \pi\delta\left(\sigma - \frac{\pi}{2}\right) \tag{2.15}$$

it follows that the mode in question corresponds to $X^L(\sigma) = -X^R(\sigma) \propto \delta(\sigma - \pi/2)$. The two delta functions cancel in taking the sum (2.9) for $X(\sigma)$. It is important to keep this over-parametrization in mind in the future.

2.2.3 String fields as operators

The formulation of Witten's cubic open string field theory action makes use of operations involving string fields, the star product $\star$ and the integration $\int$, which take a simple form when expressed in terms of half-string modes. When we regard the string field Ψ as a functional of the half-string modes $\{x^L_{2n},\ x^R_{2n}\}$ (we will use the notation $\Psi[x^L, x^R]$), the $\star$ and $\int$ operations can be written as

$$\begin{aligned} \Psi \star \Phi[x^L, x^R] &= \int [Dy]\Psi[x^L, y]\Phi[y, x^R] \\ \int \Psi &= \int [Dy]\Psi[y, y]\,, \end{aligned} \tag{2.16}$$

where $[Dy] \equiv \prod_n dy_{2n}$. These operations can also be written in operator form as follows. To any state $|\Psi\rangle$ corresponds an operator $\hat{\Psi}$ through the correspondence:

$$\begin{aligned} |\Psi\rangle &= \int [Dx^L Dx^R]\ \Psi[x^L, x^R]|x^L\rangle|x^R\rangle \\ &\Updownarrow \\ \hat{\Psi} &= \int [Dx^L Dx^R]\ \Psi[x^L, x^R]|x^L\rangle\langle x^R| \end{aligned} \tag{2.17}$$

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Such an operator formally maps a state in the "right" half-string Hilbert space to a state in the "left" half-string Hilbert space. Since these spaces can be canonically identified, we will consider $\hat{\Psi}$ to be an operator on "the" half-string Hilbert space which will be denoted by $\mathcal{H}^{\frac{1}{2}}$. We will use a subscript $_{\frac{1}{2}}$ to distinguish states in $\mathcal{H}^{\frac{1}{2}}$ from their full-string counterparts. Oscillator modes in $\mathcal{H}^{\frac{1}{2}}$ will be denoted by a superscript $^{\frac{1}{2}}$. In terms of operators on $\mathcal{H}^{\frac{1}{2}}$, star multiplication and integration reduce to operator multiplication and trace respectively:

$$\begin{aligned} \Phi \star \Psi &\Leftrightarrow \hat{\Phi}\hat{\Psi} \\ \int \Psi &\Leftrightarrow \mathrm{Tr}\,\hat{\Psi}\,. \end{aligned} \tag{2.18}$$

The hermitean inner product becomes:

$$\langle \Phi | \Psi \rangle \Leftrightarrow \mathrm{Tr}\,\hat{\Phi}^\dagger \hat{\Psi}\,. \tag{2.19}$$

In string field theory, one works with string fields for which the hermitean inner product $\langle \Phi | \Psi \rangle$ becomes equal to the BPZ inner product $\int \Phi \star \Psi$. This is guaranteed by a reality condition which becomes a Hermiticity condition in the operator formulation:

$$\Psi[x^L, x^R] = \Psi^*[x^R, x^L] \Leftrightarrow \hat{\Psi} = \hat{\Psi}^\dagger\,.$$

Summarized, under the assumption (1.1) that matter and ghost parts of the string field don't mix, the matter part of a solution to the VSFT equations of motion can be represented as an operator satisfying:

$$\begin{aligned} \hat{\Psi} &= \hat{\Psi}^\dagger \\ \hat{\Psi} &= \hat{\Psi}^2\,. \end{aligned} \tag{2.20}$$

Numerous solutions can be found by taking $\hat{\Psi}$ to be any hermitean projection operator on half-string space. These can, at least in principle, be transformed back to full string variables using the transformation formulae (2.10), (2.12). Some relevant examples will be discussed in 3.

2.2.4 Half-string creation and annihilation operators

In order to get some feeling for the meaning of the transformation to half-string modes, it is useful to see how it acts on the creation and annihilation operators. In analogy with (2.2), we define the half-string creation and annihilation operators as

$$\begin{aligned} |x^L) &= \frac{i}{\sqrt{2}} E^{ee}[|a^L) - |a^{L\dagger})] & |p^L) &= \frac{1}{\sqrt{2}}(E^{ee})^{-1}[|a^L) + |a^{L\dagger})] \\ |a^L) &= \frac{1}{\sqrt{2}}[E^{ee}|p^L) - i(E^{ee})^{-1}|x^L)] & |a^{L\dagger}) &= \frac{1}{\sqrt{2}}[E^{ee}|p^L) + i(E^{ee})^{-1}|x^L)] \end{aligned} \tag{2.21}$$

and similarly for the right half-string modes.
From (2.2), (2.21), (2.10), (2.12) we find the transformation between oscillator modes:

$$|a^e) = \frac{1}{\sqrt{2}}[|a^L) + |a^R)]$$

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$$
\begin{aligned}
|a^o) &= \frac{1}{\sqrt{2}}\Big[C^+[|a^L) - |a^R)] + C^-[|a^{L\dagger}) - |a^{R\dagger})]\Big] \\
|a^L) &= \frac{1}{\sqrt{2}}[|a^e) + C^{+T}|a^o) - C^{-T}|a^{o\dagger})] \\
|a^R) &= \frac{1}{\sqrt{2}}[|a^e) - C^{+T}|a^o) + C^{-T}|a^{o\dagger})]\,,
\end{aligned}
\tag{2.22}
$$

where the matrices $C^{\pm}$ are defined as

$$
C^{\pm} = \frac{1}{2}[E^{oo}A^T(E^{ee})^{-1} \pm (E^{oo})^{-1}A^T E^{ee}]\,.
$$

The half-string modes a^L_{2n}, $a^{L\dagger}_{2n}$, a^R_{2n}, $a^{R\dagger}_{2n}$ satisfy canonical commutation relations as can be seen from the definition (2.21) or from the properties of $C^{\pm}$:

$$
\begin{aligned}
C^{+T}C^+ - C^{-T}C^- &= 1\,, \qquad & C^+C^{+T} - C^-C^{-T} &= 1 \\
C^{+T}C^- - C^{-T}C^+ &= 0\,, \qquad & C^+C^{-T} - C^-C^{+T} &= 0\,.
\end{aligned}
\tag{2.23}
$$

2.2.5 Half-string vacuum

From (2.22) we see that, in terms of oscillators, the transformation to half-string modes is a Bogoliubov transformation under which creation and annihilation modes are mixed. Hence the vacuum $|\Omega\rangle_L|\Omega\rangle_R$ annihilated by the half-string annihilation modes is not the same as the original vacuum $|\Omega\rangle$ which is annihilated by the full-string annihilation modes. From (2.22) it follows that they are related by:

$$
\begin{aligned}
|\Omega\rangle &= \det\left(1-(C^{+-1}C^-)^2\right)^{26/4} \exp -\frac{1}{4}(a^{L\dagger} - a^{R\dagger}|C^{+-1}C^-|a^{L\dagger} - a^{R\dagger})|\Omega\rangle_L|\Omega\rangle_R \\
|\Omega\rangle_L|\Omega\rangle_R &= \det\left(1-(C^{+T-1}C^{-T})^2\right)^{26/4} \exp \frac{1}{2}(a^{o\dagger}|C^{+T-1}C^{-T}|a^{o\dagger})|\Omega\rangle\,.
\end{aligned}
\tag{2.24}
$$

The components of the matrices $C^{+-1}C^-$ and $C^{+T-1}C^{-T}$ were already calculated in [19] where they enter as Neumann coefficients in the definition of the 4-point vertex. From definition (2.40b) in [19] of the matrix $V+\bar{V}$ we see that

$$
(V+\bar{V})^{ee} = -2C^{+-1}C^- \qquad (V+\bar{V})^{oo} = -2C^{+T-1}C^{-T}\,.
$$

From equation (3.38a), the explicit matrix elements read:

$$
(C^{+T-1}C^{-T})_{2m+1,2n+1} = -\frac{\sqrt{(2m+1)(2n+1)}}{2m+1+2n+1}\binom{-\frac{1}{2}}{m}\binom{-\frac{1}{2}}{n}. \tag{2.25}
$$

It will also be useful to work out the transition between the vacua at zero momentum which we will denote by $|0\rangle_L|0\rangle_R$ and $|0\rangle$. In terms of full string modes, the state $|0\rangle_L|0\rangle_R$ satisfies:

$$
p_0|0\rangle_L|0\rangle_R = 0 \tag{2.26}
$$

$$
\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}(a_{2n+1} + a^\dagger_{2n+1})|0\rangle_L|0\rangle_R = 0 \tag{2.27}
$$

$$
a_{2m}|0\rangle_L|0\rangle_R = 0 \qquad m\neq 0 \tag{2.28}
$$

$$
\sum_{n=0}^{\infty}\left(C^{+T}_{2m,2n+1}a_{2n+1} - C^{-T}_{2m,2n+1}a^\dagger_{2n+1}\right)|0\rangle_L|0\rangle_R = 0 \qquad m\neq 0\,. \tag{2.29}
$$

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The state satisfying these conditions is given by:

$$|0\rangle_L|0\rangle_R = \det\left(1-(C^{+T-1}C^{-T})^2\right)^{26/4}\exp\frac{1}{2}(a^{o\dagger}|C^{+T-1}C^{-T}|a^{o\dagger})|0\rangle\,. \tag{2.30}$$

Conditions (2.26), (2.28), (2.29) are trivially satisfied so we only have to check (2.28). This condition is found to hold by using the identity

$$\sum_{n=0}^{\infty}\frac{1}{2n+1+a}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)} = \frac{\pi}{a}\frac{\Gamma(\frac{a+1}{2})}{\Gamma(\frac{a}{2})}$$

In [16, 24, 18], the half-string vacuum $|0\rangle_L|0\rangle_R$ is also called the "butterfly state" and the expression (6.42) in [18], derived using conformal field theory methods, agrees with (2.30).

2.2.6 Half-string Neumann coefficients

Using the relations (2.22) we can express various quantities entering in the definition of the string field theory action in terms of half-string oscillators. In principle, one could start from the full-string oscillator expressions derived in [19] and perform the Bogoliubov transformation (2.22) to half string-oscillators but this is rather cumbersome. It is more convenient to start from the original position space expressions in [1] which are naturally expressed in terms of half string modes.

The identity state in the matter sector $|\mathcal{I}^X\rangle$ defined so that $\langle\mathcal{I}^X|\Psi\rangle = \int\Psi$, has the position space expression

$$\mathcal{I}^X[x^L,x^R] = \delta(x^L-x^R)\,,$$

where $\delta(x^L-x^R)\equiv\prod_{n=0}^{\infty}\delta(x^L_{2n}-x^R_{2n})$. The oscillator expression of $|\mathcal{I}^X\rangle$ can be found by writing

$$|\mathcal{I}^X\rangle = \int[Dx^LDx^R]\mathcal{I}[x^L,x^R]|x^L\rangle|x^R\rangle$$

and using the oscillator expression for the half-string position eigenstates

$$|\{x^L_{2n}\}\rangle = N^X_{\frac{1}{2}}\exp\left[-\frac{1}{2}(x^L|(E^{ee})^{-2}|x^L)-\sqrt{2}\,i(a^{\dagger L}|(E^{ee})^{-1}|x^L)+\frac{1}{2}(a^{\dagger L}|a^{\dagger R})\right]|\Omega\rangle_L\,, \tag{2.31}$$

where

$$N^X_{\frac{1}{2}} = \left(\frac{2}{\pi}\right)^{26/4}\prod_{n=1}^{\infty}\left(\frac{2n}{\pi}\right)^{26/4}$$

(a similar expression holds for the right position eigenstates). Performing the gaussian integral one finds

$$|\mathcal{I}^X\rangle = N_{\mathcal{I}}\exp-(a^{L\dagger}|a^{R\dagger})|\Omega\rangle_L|\Omega\rangle_R \tag{2.32}$$

with

$$N_{\mathcal{I}} = \left(N^X_{\frac{1}{2}}\right)^2\det(\pi(E^{ee})^2)^{26/2} = 1.$$

The two-point vertex $|V_2^X\rangle_{12}$ is defined by ${}_{12}\langle V_2^X||\Phi\rangle_1|\psi\rangle_2 = \int\Phi\star\Psi$. Its position space representation reads:

$$V_2^X(x_1^L,x_1^R;x_2^L,x_2^R) = \delta(x_1^L-x_2^R)\delta(x_1^R-x_2^L)\,.$$

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Again using (2.31) we get

$$|V_2^X\rangle_{12} = \exp - \Big[(a_1^{L\dagger}|a_2^{R\dagger}) + (a_2^{L\dagger}|a_1^{R\dagger})\Big]\,(|\Omega\rangle_L|\Omega\rangle_R)_{12}\,. \tag{2.33}$$

Equally simple is the expression for the three-point vertex $|V_3^X\rangle_{123}$ defined by ${}_{123}\langle V_3^X||\Phi\rangle_1|\Psi\rangle_2|\Xi\rangle_3 = \int \Phi\star\Psi\star\Xi$. From the position-space expression

$$V_3^X(x_1^L, x_1^R; x_2^L, x_2^R; x_3^L, x_3^R) = \delta(x_1^L - x_3^R)\delta(x_2^L - x_1^R)\delta(x_3^L - x_2^R)$$

one gets

$$|V_3^X\rangle_{123} = \exp - \Big[(a_1^{L\dagger}|a_3^{R\dagger}) + (a_2^{L\dagger}|a_1^{R\dagger}) + (a_3^{L\dagger}|a_2^{R\dagger})\Big]\,(|\Omega\rangle_L|\Omega\rangle_R)_{123}\,. \tag{2.34}$$

The state $|V_3^X\rangle_{123}$ can also be used to calculate star products in the oscillator representation. The star product of two real string fields $|\Phi\rangle$ and $|\Psi\rangle$ is given by

$$|\Phi\star\Psi\rangle_1 = {}_2\langle\Phi|_3\langle\Psi||V_3^X\rangle_{123}. \tag{2.35}$$

As a check on the normalization of $|V_3^X\rangle_{123}$ in (2.34), one can verify that the rank one projector $|\Omega\rangle_L|\Omega\rangle_R$ indeed star-multiplies to itself under (2.35).

Comparing (2.34) with the expression in terms of full-string modes derived in [19], we see that the transformation to half-string modes has drastically simplified the Neumann matrices V^{12} and V^{21}, while the matrix V^{11} now even vanishes.

3. Some solutions to the projection equation

In this section we review some solutions to the projection equation $\Psi = \Psi\star\Psi$ which have appeared in the literature. We will restrict attention to states $|\Psi\rangle$ satisfying

$$p_0^L|\Psi\rangle = p_0^R|\Psi\rangle = 0\,. \tag{3.1}$$

Such states are invariant under separate translations of the left and right halves of the string. The solutions satisfying this condition will turn out to have a direct counterpart in the ghost sector. They can all be written as the vacuum $|0\rangle_L|0\rangle_R$ acted on with creation operators of strictly positive mode number. Note that the identity state $|\mathcal{I}\rangle$ satisfies $(p_0^L + p_0^R)|\mathcal{I}\rangle = 0$ but is not an eigenstate of $p_0^L - p_0^R$, hence it does not belong to the class of solutions we want to consider.

Of particular importance in the construction of D-brane solutions in VSFT are the projectors of rank one [8, 12]. These are constructed from any normalized state $|\chi\rangle_{\frac{1}{2}}$ in $\mathcal{H}^{\frac{1}{2}}$:

$$\hat{\Psi} = |\chi\rangle_{\frac{1}{2}\ \frac{1}{2}}\langle\chi|\,.$$

Their position-space form is given by

$$\Psi[x^L, x^R] = \chi[x^L]\chi^*[x^R]$$

and the corresponding Fock space state is

$$|\Psi\rangle = |\chi\rangle_L|\chi^*\rangle_R$$

where $|\chi^*\rangle$ denotes the state with wavefunctional $\chi[x]^*$.

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3.1 Half-string vacuum

The simplest example of a rank one projector is provided by taking $|\chi\rangle_{\frac{1}{2}} = |0\rangle_{\frac{1}{2}}$. The corresponding solution is the half-string vacuum or butterfly state $|B\rangle = |0\rangle_L|0\rangle_R$, whose full-string form was given in in (2.30):

$$|B\rangle = |0\rangle_L|0\rangle_R = \det\left(1-(C^{+T-1}C^{-T})^2\right)^{26/4} \exp -\frac{1}{2}(a^{o\dagger}|C^{+T-1}C^{-T}|a^{o\dagger})|\Omega\rangle\,.$$

3.2 D25-brane sliver

This solution, which describes a single D25-brane, was found initially [5] in full-string variables where it takes the form of a squeezed state

$$|\Sigma\rangle = \det(1-S^2)^{26/4} \exp -\frac{1}{2}((a^\dagger|S|a^\dagger))|0\rangle$$

here we have introduced a new notation for sums that exclude the zero mode: $((a|b)) \equiv \sum_{n=1}^{\infty} a_n b_n$. The matrix S is related to the Neumann matrix V^{11} [19] by:

$$CS = \frac{1}{2X}\left(1+X-\sqrt{(1+3X)(1-X)}\right), \tag{3.2}$$

where C is the twist matrix $C_{mn} = (-1)^n\delta_{mn}$ and $X = CV^{11}$. The sliver state $|\Sigma\rangle$ was shown to satisfy (3.1) in [22]. Evidence for the fact that it corresponds to a rank one projector in half-string space was found in [8] and a proof was given in [25]. The corresponding half-string state $|\chi_\Sigma\rangle_{\frac{1}{2}}$ is again a squeezed state:

$$|\chi_\Sigma\rangle_{\frac{1}{2}} \propto \exp -\frac{1}{2}((a^{\frac{1}{2}\dagger}|\frac{1-D}{1+D}|a^{\frac{1}{2}\dagger}))|0\rangle_{\frac{1}{2}}\,.$$

The D25-brane sliver can be seen as the projector on the vacuum for a set of half-string oscillators related to the original ones by a Bogoliubov transformation. The matrix D can be expressed in terms of previously defined matrices using (2.30-2.31) in [25]. One finds

$$D = \tilde{E}^{ee}\tilde{A}(E^{oo})^{-1}\sqrt{E^{oo}\tilde{A}^T(\tilde{E}^{ee})^{-2}\tilde{A}E^{oo}}(E^{oo})^{-1}\tilde{A}^T\tilde{E}^{ee}\,,$$

where a $\tilde{}$ denotes the submatrix obtained by excluding the zero mode. One should be careful in trying to simplify this expression since operators like $\sqrt{\tilde{E}^{ee}\tilde{A}(E^{oo})^{-1}}$ are ill-defined.

3.3 GRSZ projectors

In [18], a class of rank one projectors, including the D25-brane sliver and the half-string vacuum as special cases, was constructed using CFT methods. These arise from surface states in CFT and they all satisfy the condition (3.1). In terms of oscillators, all these projectors are squeezed states of the form

$$|\Xi\rangle = \det(1-V^2)^{26/4} \exp -\frac{1}{2}((a^\dagger|V|a^\dagger))|0\rangle\,.$$

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The condition (3.1) implies that the vector v defined by

$$v_{2n+1} = \frac{(-1)^n}{\sqrt{2n+1}}; \qquad v_{2n} = 0$$

is an eigenvector of V with eigenvalue 1. The simplest example of this construction is provided by taking V to be the identity matrix. The corresponding projector $|\mathcal{N}\rangle$ is called the *nothing state* in [18]:

$$|\mathcal{N}\rangle = \exp -\frac{1}{2}((a^\dagger|a^\dagger))|0\rangle\,.$$

The corresponding half-string state $|\chi_N\rangle_{\frac{1}{2}}$ is given by:

$$|\chi_{\mathcal{N}}\rangle_{\frac{1}{2}} = \exp -\frac{1}{2}((a^{\frac{1}{2}\dagger}|a^{\frac{1}{2}\dagger}))|0\rangle_{\frac{1}{2}}\,.$$

4. Vacuum string field theory: ghost sector

4.1 Bosonization conventions

We will work in the bosonized formalism in which the b, c ghosts are be expressed in terms of a scalar linear dilaton field $\phi(\sigma)$ [26, 1]. The mode expansion is

$$\phi(\sigma) = \phi_0 + \sqrt{2}\sum_{n=1}^{\infty}\phi_n\cos(n\sigma)\,. \tag{4.1}$$

and similarly for the conjugate momentum $\pi(\sigma)$:

$$\pi(\sigma) = \frac{1}{\pi}\left(\pi_0 + \sqrt{2}\sum_{n=1}^{\infty}\pi_n\cos(n\sigma)\right). \tag{4.2}$$

with

$$[\phi_n, \pi_m] = i\delta_{mn}\,.$$

The momentum zero-mode π_0 plays the role of the ghost number and is quantized in half-integer units. The string field Ψ entering in the VSFT action has ghost number $\pi_0 = -1/2$ while the string field parametrizing gauge transformations has $\pi_0 = -3/2$. Creation and annihilation operators d_n, d_n^* are defined by

$$\begin{aligned} |\phi) &= \frac{i}{\sqrt{2}}E[|d) - |d^*)] & |\pi) &= \frac{1}{\sqrt{2}}E^{-1}[|d) + |d^*)] \\ |d) &= \frac{1}{\sqrt{2}}[E|\pi) - iE^{-1}|\phi)] & |d^*) &= \frac{1}{\sqrt{2}}[E|\pi) + iE^{-1}|\phi)] \end{aligned} \tag{4.3}$$

with commutation relations:

$$[d_m, d_n^*] = \delta_{mn}\,.$$

The ghost fields $b_\pm$, $c^\pm$ with mode expansions

$$\begin{aligned} c^\pm(\sigma) &= \sum_{n\in\mathbb{Z}} c_n e^{\pm in\sigma} \\ b_\pm(\sigma) &= \sum_{n\in\mathbb{Z}} b_n e^{\pm in\sigma} \end{aligned} \tag{4.4}$$

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are bosonized according to:

$$
\begin{aligned}
c^+(\sigma) &= :e^{i\phi^+(\sigma)}: := e^{i\phi_0}e^{i\sigma(\pi_0+1/2)}e^{\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}e^{in\sigma}d_n^*}e^{-\sum_{m=1}^{\infty}\frac{1}{\sqrt{m}}e^{-im\sigma}d_m} \\
c^-(\sigma) &= :e^{i\phi^-(\sigma)}: := e^{i\phi_0}e^{-i\sigma(\pi_0+1/2)}e^{\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}e^{-in\sigma}d_n^*}e^{-\sum_{m=1}^{\infty}\frac{1}{\sqrt{m}}e^{im\sigma}d_m} \\
b_+(\sigma) &= :e^{-i\phi^+(\sigma)}: := e^{-i\phi_0}e^{-i\sigma(\pi_0-1/2)}e^{-\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}e^{in\sigma}d_n^*}e^{\sum_{m=1}^{\infty}\frac{1}{\sqrt{m}}e^{-im\sigma}d_m} \\
b_-(\sigma) &= :e^{-i\phi^-(\sigma)}: := e^{-i\phi_0}e^{i\sigma(\pi_0-1/2)}e^{-\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}e^{-in\sigma}d_n^*}e^{\sum_{m=1}^{\infty}\frac{1}{\sqrt{m}}e^{im\sigma}d_m}\,.
\end{aligned}
\tag{4.5}
$$

Hermitean conjugation works on the ghost modes as $c_n^\dagger = c_{-n}$, so the ghost fields $c^\pm(\sigma), b_\pm(\sigma)$ are hermitean. From (4.5) it follows that hermitean conjugation acts on the bosonized ghosts modes as:

$$
d_n^\dagger = -d_n^*\,; \qquad \phi_n^\dagger = -\phi_n\,; \qquad \pi_n^\dagger = -\pi_n \tag{4.6}
$$

This leads to slight differences with the matter sector so we proceed to give some further conventions in the ghost sector. As before, we define a vacuum $|\Omega\rangle$ satisfying $d_n|\Omega\rangle = 0$, and its hermitean conjugate $\langle\Omega| = (|\Omega\rangle)^\dagger$ satisfying $\langle\Omega|d_n^* = 0$ and $\langle\Omega|\Omega\rangle = 1$. Position eigenstates have the oscillator expression

$$
|\phi\rangle \equiv |\{\phi_n\}\rangle = N^\phi \exp\left[-\frac{1}{2}(\phi|E^{-2}|\phi) - \sqrt{2}\,i(d^*|E^{-1}|\phi) + \frac{1}{2}(d^*|d^*)\right]|\Omega\rangle\,, \tag{4.7}
$$

with $N^\phi = (N^X)^{1/26}$, while their hermitean conjugates read

$$
\langle\phi| \equiv (|\phi\rangle)^\dagger = N^\phi\langle\Omega|\exp\left[-\frac{1}{2}(\phi|E^{-2}|\phi) - \sqrt{2}\,i(d|E^{-1}|\phi) + \frac{1}{2}(d|d)\right]. \tag{4.8}
$$

These satisfy

$$
\langle\phi|\hat{\phi}_n = \langle\phi|(-\phi_n)\,,
$$

where $\hat{\phi}_n$ refers to the operator and ϕ_n to its eigenvalue. The inner product reads

$$
\langle\phi'|\phi\rangle = \delta(\phi+\phi')\,.
$$

Hence one finds the completeness relation

$$
\mathbf{1} = \int [D\phi]|\phi\rangle\langle-\phi|\,.
$$

Similarly, for momentum eigenstates one has

$$
\langle\pi|\hat{\pi}_n = \langle\pi|(-\pi_n)\,; \qquad \langle\pi'|\pi\rangle = \delta(\pi+\pi')\,.
$$

One goes to the position-space representation by expanding

$$
|\Psi\rangle = \int [D\phi]\Psi[\phi]|\phi\rangle\,,
$$

where

$$
\Psi[\phi] = \langle-\phi|\Psi\rangle\,.
$$

The inner product can be written as

$$
\langle\Phi|\Psi\rangle = \int [D\phi]\Phi^*[-\phi]\Psi[\phi]\,.
$$

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4.2 Half-string modes

The formulae for the transition to half-string modes derived for the matter sector go through for the ghost sector as well under the substitutions:

$$x \to \phi \qquad p \to \pi \qquad a \to d\,, \qquad a^\dagger \to d^*\,.$$

Half-string modes are defined by

$$\begin{aligned} |\phi^L) &= \frac{1}{\sqrt{2}}[|\phi^e) + A|\phi^o)] \qquad & |\phi^R) &= \frac{1}{\sqrt{2}}[|\phi^e) - A|\phi^o)] \\ |\pi^L) &= \frac{1}{\sqrt{2}}[|\pi^e) + A|\pi^o)] \qquad & |\pi^R) &= \frac{1}{\sqrt{2}}[|\pi^e) - A|\pi^o)]\,. \end{aligned} \tag{4.9}$$

While the transformation between oscillator modes reads:

$$\begin{aligned} |d^e) &= \frac{1}{\sqrt{2}}[|d^L) + |d^R)] \\ |d^o) &= \frac{1}{\sqrt{2}}\left[C^+[|d^L) - |d^R)] + C^-[|d^{L\dagger}) - |d^{R\dagger})]\right] \\ |d^L) &= \frac{1}{\sqrt{2}}[|d^e) + C^{+T}|d^o) - C^{-T}|d^{o\dagger})] \\ |d^R) &= \frac{1}{\sqrt{2}}[|d^e) - C^{+T}|d^o) + C^{-T}|d^{o\dagger})] \end{aligned} \tag{4.10}$$

Full- and half-string vacua are related through

$$|\Omega\rangle_L|\Omega\rangle_R = \det\left(1-(C^{+T-1}C^{-T})^2\right)^{1/4} \exp -\frac{1}{2}(d^{o*}|C^{+T-1}C^{-T}|d^{o*})|\Omega\rangle\,.$$

4.3 Vertices and midpoint insertions

The states defining the one- and three-point vertices in the ghost sector differ from the pure delta-function overlaps of the matter sector through the presence of midpoint insertions. These have their origin in the fact that these vertices introduce delta-function curvature on the string world-sheet to which the linear dilaton couples. Let us first introduce states $|\mathcal{I}^\phi\rangle$, $|V_2^\phi\rangle_{12}$ and $|V_3^\phi\rangle_{123}$ defining pure delta-function overlaps analogous to the ones used in the matter sector:

$$\begin{aligned} \mathcal{I}^\phi(\phi^L,\phi^R) &= \delta(\phi_1^L - \phi_2^R) \\ V_2^\phi(\phi_1^L,\phi_1^R;\phi_2^L,\phi_2^R) &= \delta(\phi_1^L - \phi_2^R)\delta(\phi_1^R - \phi_2^L) \\ V_3^\phi(\phi_1^L,\phi_1^R;\phi_2^L,\phi_3^R;\phi_2^L,\phi_3^R) &= \delta(\phi_1^L - \phi_3^R)\delta(\phi_2^L - \phi_1^R)\delta(\phi_3^L - \phi_2^R)\,. \end{aligned} \tag{4.11}$$

In terms of oscillators, these are given by (cfr. (2.32), (2.33), (2.34))

$$\begin{aligned} |\mathcal{I}^\phi\rangle &= \exp -(d^{L*}|d^{R*})|\Omega\rangle_L|\Omega\rangle_R \\ |V_2^\phi\rangle_{12} &= \exp -[(d_1^{*L}|d_2^{*R}) + (d_2^{*L}|d_1^{*R})](|\Omega\rangle_L|\Omega\rangle_R)_{12} \\ |V_3^\phi\rangle_{123} &= \exp -[(d_1^{*L}|d_3^{*R}) + (d_2^{*L}|d_1^{*R}) + (d_3^{*L}|d_2^{*R})](|\Omega\rangle_L|\Omega\rangle_R)_{123}\,. \end{aligned} \tag{4.12}$$

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The ghost sector vertices $|\mathcal{I}^{gh},\ \rangle|V_2^{gh}\rangle_{12}$ and $|V_3^{gh}\rangle_{123}$ include midpoint insertions:

$$\begin{aligned} |\mathcal{I}^{gh}\rangle &= e^{-\frac{3}{2}i\phi(\pi/2)}|\mathcal{I}^{\phi}\rangle \\ |V_2^{gh}\rangle_{12} &= |V_2^{\phi}\rangle_{12} \\ |V_3^{gh}\rangle_{123} &= e^{\frac{i}{2}(\phi_1(\pi/2)+\phi_2(\pi/2)+\phi_3(\pi/2))}|V_3^{\phi}\rangle_{123}\,. \end{aligned} \tag{4.13}$$

In the last expression, we have to chosen to divide the midpoint insertion evenly over the three copies of the ghost Fock space labeled by $_1$, $_2$, $_3$. This is possible because of the delta-function character of V_3^{ϕ}. Note that, by making a field redefinition, one could absorb the midpoint insertions of the three-point vertex in the definition of the string field, but this would introduce a midpoint insertion in the two-point vertex. Hence it seems hard to get rid of the midpoint insertions in the string field theory action. However, we will see that if the quadratic term in the action contains the BRST operator proposed in [16], which is itself a midpoint insertion, it will be possible to effectively get rid of all the midpoint insertions in the action through a field redefinition.

4.4 BRST operator

In [16], it was proposed that the BRST operator governing the VSFT action is a pure midpoint insertion:

$$Q = \frac{1}{2}\left[c^+\left(\frac{\pi}{2}\right) + c^-\left(\frac{\pi}{2}\right)\right].$$

Using (4.5) and the Baker-Campbell-Hausdorff formula, we can rewrite Q in the bosonized form

$$Q = \frac{\kappa_1}{2}e^{i\phi(\pi/2)}\left[e^{i\frac{\pi}{2}(\pi_0+\frac{1}{2})}e^{i\sqrt{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\pi_{2n+1}} + e^{-i\frac{\pi}{2}(\pi_0+\frac{1}{2})}e^{-i\sqrt{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\pi_{2n+1}}\right], \tag{4.14}$$

where κ_1 is an (infinite) constant: $\kappa_1 = \exp\sum_{n=1}\frac{1}{2n}$. This constant can be absorbed into a redefinition of the string field and the overall normalization of the action. By definition, the states entering the string field theory action have fixed ghost number $\pi_0 = -1/2$. The factors $e^{\pm i\frac{\pi}{2}(\pi_0+\frac{1}{2})}$ in (4.14) act trivially on such states. The remaining dependence of Q on the momentum modes becomes very simple in terms of half-string modes: using (4.9), (2.5) we have

$$\sqrt{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\pi_{2n+1} = \frac{\pi}{2\sqrt{2}}(\pi_0^L - \pi_0^R)\,.$$

Hence, when acting on states with $\pi_0 = -1/2$, Q is equivalent to the operator $\kappa_1 e^{i\phi(\pi/2)}Q'$ where, for later convenience, we have defined

$$Q' = \cos\frac{\pi}{2\sqrt{2}}(\pi_0^L - \pi_0^R)\,. \tag{4.15}$$

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5. Ghost sector projection equation

5.1 Field redefinition

In this section we propose a field redefinition which considerably simplifies the equation of motion in the ghost sector of VSFT. This equation arises from the action

$$S^{gh}[\Psi] = -\kappa_0 \left[\frac{1}{2}\, {}_{12}\langle V_2^{gh}||\Psi\rangle_1 Q|\Psi\rangle_2 + \frac{1}{3}\, {}_{123}\langle V_3||\Psi\rangle_1|\Psi\rangle_2|\Psi\rangle_3 \right] \tag{5.1}$$

with κ_0 an overall normalization constant. Using the results of the previous section, the quadratic term can be rewritten as

$$\frac{1}{2}\, {}_{12}\langle V_2^{gh}||\Psi\rangle_1 Q|\Psi\rangle_2 = \frac{\kappa_1}{2}\, {}_{12}\langle V_2^{\phi}||\Psi\rangle_1 e^{i\phi(\pi/2)} Q'|\Psi\rangle_2 \,.$$

We can use the fact that $|V_2^{\phi}\rangle$ satisfies the overlap equations

$$(\phi_{1,2n} - \phi_{2,2n})|V_2^{\phi}\rangle_{12} = 0 \tag{5.2}$$

(this can be derived, e.g., from the oscillator expression (4.12)), to evenly distribute the $e^{i\phi(\pi/2)}$ insertion over the two copies of the ghost Fock space labeled by ${}_1$ and ${}_2$:

$$\begin{aligned} \frac{1}{2}\, {}_{12}\langle V_2^{gh}||\Psi\rangle_1 Q|\Psi\rangle_2 &= \frac{\kappa_1}{2}\, {}_{12}\langle V_2|e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_1 e^{\frac{i}{2}\phi(\pi/2)} Q'|\Psi\rangle_2 \\ &= \frac{\kappa_1}{2}\, {}_{12}\langle V_2|e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_1 Q' e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_2 \end{aligned} \tag{5.3}$$

where, in the second line, we used the fact that $[\phi(\pi/2), (\pi_0^L - \pi_0^R)] = 0$. Using the definition (4.13) of the three-point vertex, the action (5.1) reduces to

$$\begin{aligned} S^{gh}[\Psi] = -\kappa_0 \Bigg[&\frac{\kappa_1}{2}\, {}_{12}\langle V_2^{\phi}|e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_1 Q' e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_2 + \\ &+ \frac{1}{3}\, {}_{123}\langle V_3^{\phi}|e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_1 e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_2 e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle_3 \Bigg] . \end{aligned} \tag{5.4}$$

We can now absorb the midpoint insertions $e^{\frac{i}{2}\phi(\pi/2)}$ into a field redefinition

$$|\Psi'\rangle = -\frac{1}{\kappa_1} e^{\frac{i}{2}\phi(\pi/2)}|\Psi\rangle \,. \tag{5.5}$$

We get

$$S^{gh}[\Psi'] = -\kappa_0' \left[\frac{1}{2}\, {}_{12}\langle V_2^{\phi}| \left(|\Psi'\rangle_1 Q'|\Psi'\rangle_2\right) - \frac{1}{3}\, {}_{123}\langle V_3^{\phi}| \left(|\Psi'\rangle_1|\Psi'\rangle_2|\Psi'\rangle_3\right) \right] ,$$

where $\kappa_0' = \kappa_0(\kappa_1)^3$. The equation of motion becomes:

$$Q'|\Psi'\rangle = |\Psi'\rangle \star' |\Psi'\rangle \,, \tag{5.6}$$

where $\star'$ is determined by the vertex $|V_3^{\phi}\rangle$ without midpoint insertions and has the same form as the star product (2.35) in the matter sector.

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Since $|\Psi\rangle$ was restricted to have ghost number $-1/2$, the new string field $|\Psi'\rangle$ is constrained to have ghost number zero:

$$(\pi_0^L + \pi_0^R)|\Psi'\rangle = 0\,. \tag{5.7}$$

When we restrict attention to states which in addition are eigenstates of π_0^L and π_0^R separately, i.e. states satisfying

$$\pi_0^L|\Psi'\rangle = -\pi_0^R|\Psi'\rangle = \lambda|\Psi'\rangle \tag{5.8}$$

for some λ, the action of the operator Q' in (5.6) reduces to multiplication by a constant. Note that, although the full-string ghost number π_0 was quantized, there is no a priori restriction on the value of the half-string ghost number λ. We should, however, make sure that the restriction (5.8) is compatible with the reality condition on the string field, which in the ghost sector reads [1]

$$\Psi'[\phi^L, \phi^R] = \Psi'^*[-\phi^R, -\phi^L]\,. \tag{5.9}$$

This restricts the allowed values of λ to $\lambda = 0$. Indeed, for general λ, the zero-mode part of $\Psi'[\phi^L, \phi^R]$ is $e^{i\lambda(\phi_0^L - \phi_0^R)}$. Hence the reality condition is satisfied only for $\lambda = 0$.

In summary, a class of solutions to the equations of motion in the ghost sector of VSFT is provided by states $|\Psi'\rangle$ that satisfy the condition

$$\pi_0^L|\Psi'\rangle = \pi_0^R|\Psi'\rangle = 0 \tag{5.10}$$

and are solutions of the projection equation

$$|\Psi'\rangle = |\Psi'\rangle \star' |\Psi'\rangle\,. \tag{5.11}$$

In addition, they should satisfy the reality condition (5.9).

The condition (5.10) can be interpreted as stating that the ghost number of $|\Psi'\rangle$ (and hence the ghost number of the original field $|\Psi\rangle$ as well) should be evenly distributed over the left- and right halves of the string.

5.2 Operator formulation

The absence of midpoint insertions in $\star'$ implies that we can write (5.11) in the form of a simple algebraic equation for operators in half-string space, just as in the matter case. The map between string fields $|\Psi'\rangle$ and half-string operators $\hat{\Psi}'$ is slightly different from the one in the matter sector because ϕ is antihermitean. It reads:

$$\begin{aligned} |\Psi'\rangle &= \int [D\phi^L D\phi^R]\, \Psi'[\phi^L, \phi^R]|\phi^L\rangle\phi^R\rangle \\ &\Updownarrow \\ \hat{\Psi}' &= \int [D\phi^L D\phi^R]\, \Psi'[\phi^L, \phi^R]|\phi^L\rangle\langle -\phi^R|\,. \end{aligned} \tag{5.12}$$

With this definition, $\star'$ multiplication reduces to ordinary multiplication of operators:

$$\Phi' \star' \Psi' \Leftrightarrow \hat{\Phi}'\hat{\Psi}'\,.$$

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The reality condition on the string field (5.9) again reduces to a Hermiticity condition on operators:

$$\Psi'[\phi^L, \phi^R] = \Psi'^*[-\phi^R, -\phi^L] \Leftrightarrow \hat{\Psi}' = \hat{\Psi}'^\dagger \,.$$

The operator form of the conditions (5.9), (5.10) and the equation of motion (5.11) is

$$\hat{\Psi}' = \hat{\Psi}'^\dagger \tag{5.13}$$

$$\pi_0^{\frac{1}{2}}\ \hat{\Psi}' = \hat{\Psi}'\,\pi_0^{\frac{1}{2}} = 0 \tag{5.14}$$

$$\hat{\Psi}' = \hat{\Psi}'^2 \,. \tag{5.15}$$

The ghost number condition (5.14), together with Hermiticity (5.13), restricts the zero mode part of $\hat{\Psi}'$ to be $|\pi_0 = 0\rangle_{\frac{1}{2}\ \frac{1}{2}}\langle\pi_0 = 0|$. Hence we can decompose

$$\hat{\Psi}' = \Big(|\pi_0 = 0\rangle_{\frac{1}{2}\ \frac{1}{2}}\langle\pi_0 = 0|\Big) \otimes \hat{M} \,. \tag{5.16}$$

where $\hat{M}$ works on the subspace orthogonal to the zero mode. The equations (5.13), (5.15) simply state that $\hat{M}$ is a hermitean projection operator.

With the ghost zero-mode part constrained as in (5.16), VSFT becomes, at least formally, equivalent to a cubic U(∞) matrix model. Indeed, if we define the operator $\hat{P}$ as the tensor product of $\hat{M}$ with the matter string field Ψ^m, $\hat{P} = \hat{\Psi}^m \otimes \hat{M}$, the total matter and ghost action becomes

$$S[\hat{P}] = -\kappa_0' \,\mathrm{Tr}\left[\frac{1}{2}\hat{P}^2 - \frac{1}{3}\hat{P}^3\right]$$

which is invariant under the unitary transformations $\hat{P} \to U\hat{P}U^\dagger$. Here, U works on the half-string subspace orthogonal to the ghost zero mode. A similar U(∞) invariance is present in purely cubic string field theory [27].

5.3 Examples

It is now a simple matter to construct ghost sector solutions. Using the decomposition (5.16) any hermitean projection operator $\hat{M}$ in the subspace of $\mathcal{H}^{\frac{1}{2}}$ not involving the zero mode will give rise to a ghost sector solution. In section 3, we gave several examples of rank one projectors in the matter sector with half-string momenta equal to zero. These can all be mapped to projectors in the ghost sector satisfying (5.14), (5.15). Their wavefunctionals factorize into a product of half-string gaussians so they also obey the reality condition (5.9).

A first example is the *ghost butterfly state*

$$|B'\rangle = 0\rangle_L|0\rangle_R = \det\left(1 - (C^{+T-1}C^{-T})^2|\right)^{1/4} \exp -\frac{1}{2}(d^{o*}|C^{+T-1}C^{-T}|d^{o*})|0\rangle \,.$$

The matrix elements of $C^{+T-1}C^{-T}$ were given in (2.25).

From the D25-brane sliver in the matter sector one can construct an analogous *ghost sliver state* given by

$$|\Sigma'\rangle = \det(1 - S^2)^{1/4} \exp -\frac{1}{2}((d^*|S|d^*))|0\rangle \,.$$

with the matrix S given by (3.2).

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All projectors arising from surface states constructed in [18] have analogous solutions in the ghost sector. The simplest example is the *ghost sector nothing state* given by

$$|\mathcal{N}'\rangle = \exp -\frac{1}{2}((d^*|d^*))|0\rangle\,.$$

Except for the ghost sector nothing state, which is not normalizable, the solutions discussed here are rank one projectors for which the ghost part of the action takes the value $-\kappa_0'/6$. If we combine such a rank one ghost projector with a rank one matter solution, the total action will still be equal to $-\kappa_0'/6$. This suggests the identification

$$\kappa_0'/6 = \mathcal{T}_{25}\,,$$

where $\mathcal{T}_{25}$ is the tension of the D25-brane.

The fact that we find finite action solutions for finite values of κ_0' seems in contradiction with [16], where it was found that the analogous constant $\kappa_0^{\rm GRSZ}$ should diverge in order to have finite action solutions. However, one has to bear in mind that the normalization of the vertices in (4.13) in the string field theory action differs from the one used in [16] by singular factors [13, 25]. A more precise derivation of the relation between κ_0' and $\kappa_0^{\rm GRSZ}$ would require a careful regularization of such factors and is beyond the scope of the present work.

6. Discussion

In this paper we have focused on the ghost sector of VSFT with the BRST-operator Q taken to be the pure midpoint insertion proposed in [16]. We have shown that, for this particular choice of Q, the main obstacles to constructing ghost sector solutions, viz. the presence of Q in the equations of motion and the midpoint insertion in the star product, can be dealt with simultaneously by making a field redefinition. This redefinition removes the midpoint insertion from the star product and the remaing kinetic operator Q' depends, when expressed in half-string modes, only on the difference of the left- and right ghost numbers. Hence it can be rendered trivial by consistently truncating the string field to have its ghost number symmetrically divided over the left- and right halves of the string. The resulting equation of motion is a pure projection equation and solutions can be found by using the techniques developed for the matter sector. In the light of certain subtleties in the transition to half-string variables, hinted at in section 2.2.2 and discussed more extensively in [22, 21, 23], we should note that the use of half-string variables is not essential in our construction: it merely provides a simple interpretation of the operator Q' in terms of half string zero modes and facilitates the construction of solutions as projection operators on half-string Hilbert space. We end with some observations and open problems.

- It seems that the relation between the ghost sector equation of motion and the projection equation is a feature special to the pure midpoint BRST-operator Q in contrast to the other pure ghost operators considered previously [4]. Indeed, it is unlikely that similar simplifications would go through with any of the other pure ghost BRST operators. It is quite remarkable that precisely this Q has emerged as a likely candidate for the correct description of the ghost sector. It has been remarked in [16]

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that non-pure ghost BRST operators, obtained by tensoring Q with a suitable matter part, could also be considered good candidates for describing VSFT. A generalization along these lines appears, in fact, to be necessary in order to obtain the correct structure of gauge transformations in VSFT [28]. The simplifications in the ghost sector discussed in this paper would also go through for these operators.

- In this paper, we used the bosonized form of the ghost system. It remains to be seen whether a similar approach would work in terms of fermionic ghosts since the field redefinition (5.5) has no local expression in terms of (b, c) fields.

- In [16], solutions in the ghost sector of VSFT were constructed by introducing new 'twisted ghost' variables, in terms of which these solutions reduced to pure projectors. The transition to twisted variables acts on the Virasoro generators as

$$L_n^{tw} = L_n + n j_n + \delta_{n,0}$$

with j_n the modes of the ghost number current. One might wonder how this approach is related to the one used in the present paper. A first guess would be that our field redefinition (5.5) implements the transformation to the twisted ghost system:

$$L_n^{tw} \stackrel{?}{=} L'_n \equiv e^{\frac{i}{2}\phi(\pi/2)} L_n e^{-\frac{i}{2}\phi(\pi/2)}$$

This is not the case however as one can easily verify. It seems likely that both approaches are related but it is not yet clear how.

- In section 5.2, we found that, once one imposes the constraint (5.10) on the half-string ghost number, VSFT can be written as a U(∞) matrix model. Such a U(∞) symmetry was also also argued to be present in the tachyonic vacuum from the low-energy point of view in the presence of a large B-field [15]. It would be interesting to see how both points of view are related.

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